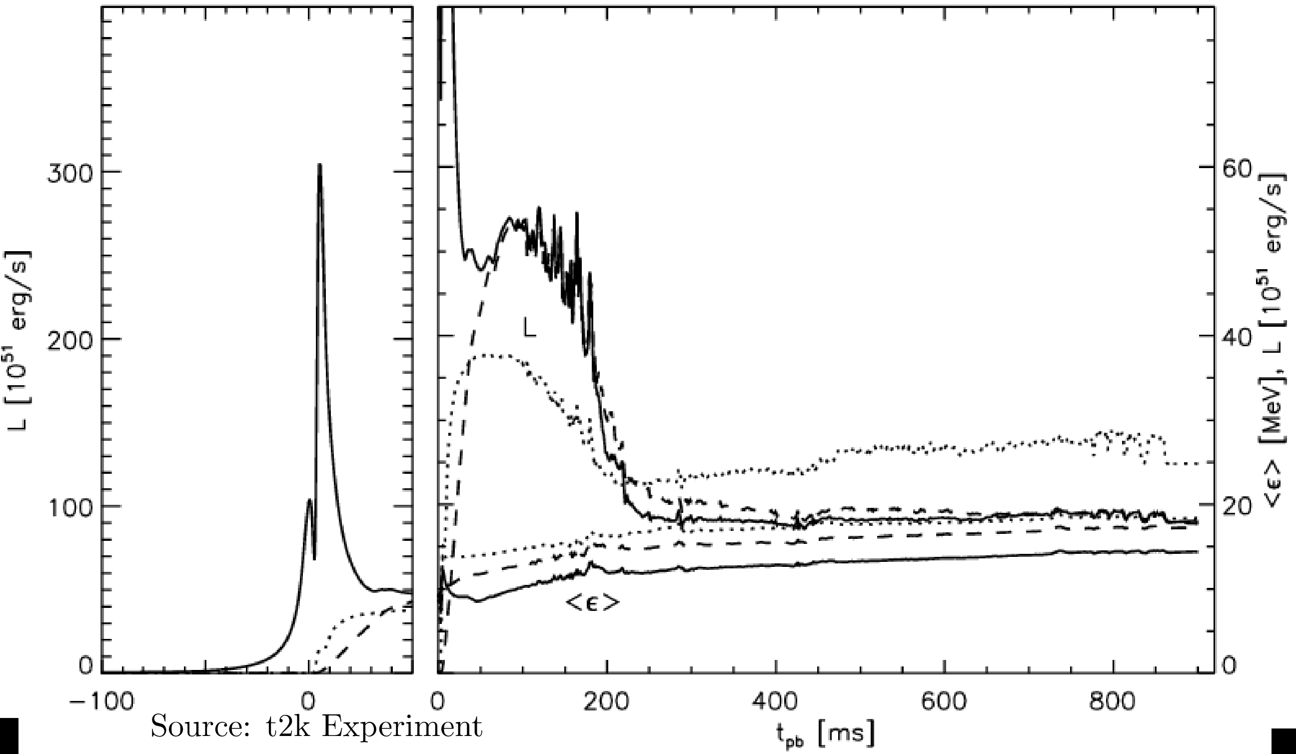
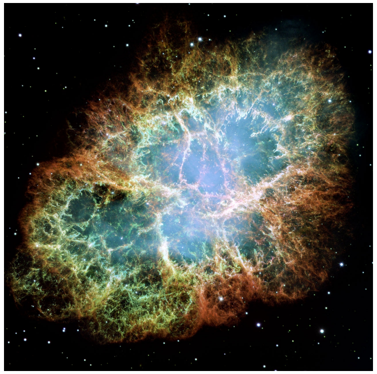
Neutrino Transport in Core-Collapse Supernovae

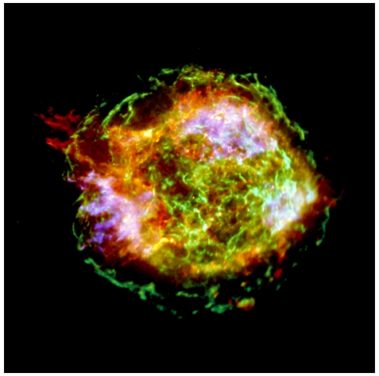


Source: t2k Experiment



Crab Nebula (Hubble Space Telescope)

Master's Presentation Talk
Paul Mekhedjian
October 23rd, 2014



Cassiopeia A (Chandra X-Ray Observatory)

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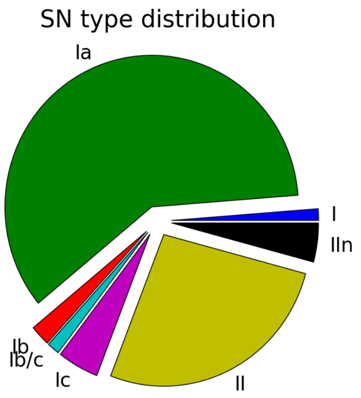
Talk Outline

- Introduction & Background
- Goals & Methods
- Results & Analysis
- Summary & Conclusion

Core Collapse Death of Massive Stars

- Focus on Type II SNe:



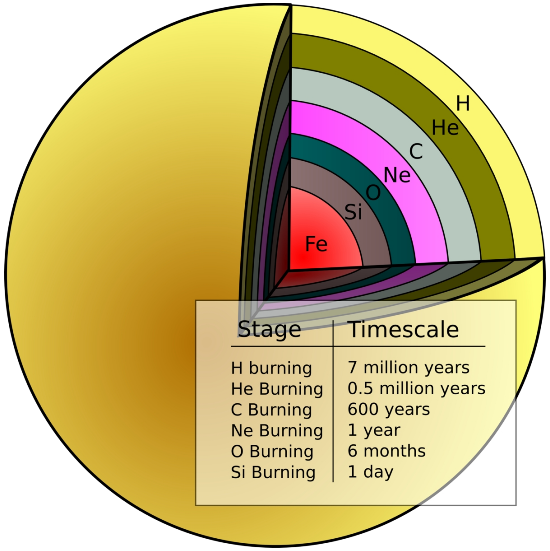
Source: Padova-Asiago Supernova Group

Type Ia: Thermonuclear explosion
Types II, Ib/Ic: Core Collapse

- Chandrasekhar mass defines maximum stable mass supported by degenerate electron pressure:

$$M_{\mathrm{ch}} \simeq 1.457 \cdot \left(\frac{Y_{\mathrm{e}}}{0.5}\right)^2 \cdot M_\odot$$

- Burning at exponential timescales...



Stage	Timescale
H burning	7 million years
He Burning	0.5 million years
C Burning	600 years
Ne Burning	1 year
O Burning	6 months
Si Burning	1 day

$$M \gtrsim 8 M_\odot$$

Source: A. C. Phillips, *The Physics of Stars*

- Iron core conditions right before / at collapse:

$$\rho_c \simeq 10^9 - 10^{10}\ \mathrm{g/cm^3}$$

$$\mathrm{T}_c \simeq (8-10) \times 10^9 K$$

$$\mathrm{t}_{\mathrm{collapse}} \simeq \frac{1}{\sqrt{\mathrm{G} \cdot \rho_c}} \longrightarrow t_{\mathrm{collapse}} \approx 100\ \mathrm{ms}$$

Collapse Dynamics

1. As ρ_c increases, so does μ_e: $\mu_e(\rho) \propto \rho^{1/3}$

$$e^- + p \longrightarrow n + \nu_e$$

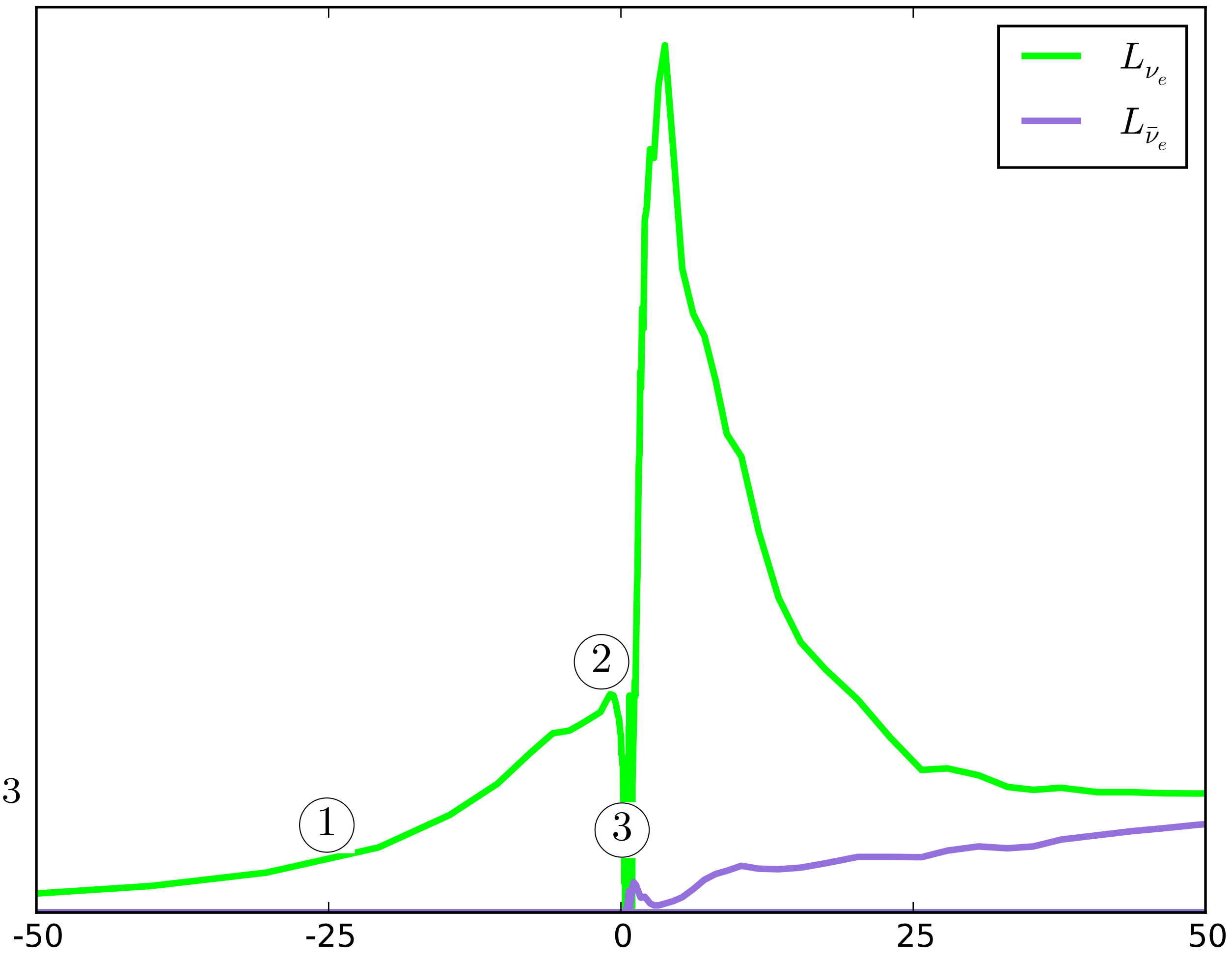
Electron capture on bound protons lowers Y_e and consequently M_ch and collapse of iron core ensues.

2. At $\rho \gtrsim 2 \times 10^{12}\ \mathrm{g/cm}^3$ neutrinos begin to become trapped:

$$\tau_\mathrm{diffusion} \gtrsim \tau_\mathrm{collapse}$$

3. Nuclear saturation density at $\rho \simeq 2.7 \times 10^{14}\ \mathrm{g/cm}^3$ leads to stronger neutrino diffusion, core bounce and shock:

$$\mathrm{E}^{init}_\mathrm{shock} \approx (4 - 10) \times 10^{51}\ \mathrm{erg}$$



Luminosity data from Agile-IDSA

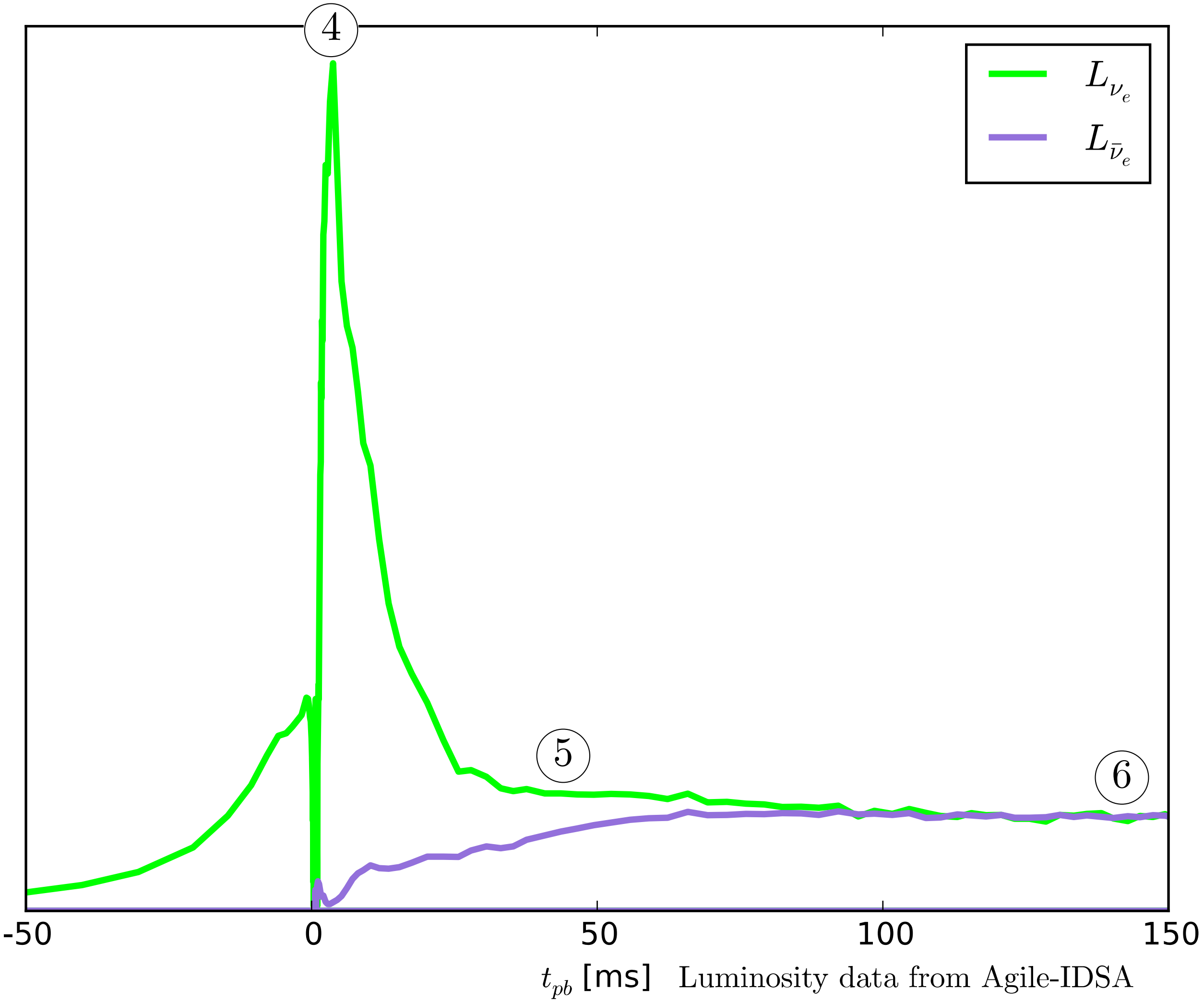
Neutrino Burst, Accretion and Heating

4. Neutrino burst at ~ 3 ms post-bounce. Deleptonization and ν-burst weaken shock. Photo-dissociation of baryonic matter eventually stops it *(prompt shock stalling)*.

5. Gravitational energy from accretion fuels added neutrino luminosity: $\mathrm{L}_\nu^{\mathrm{acc}}$

$\mathrm{T}\uparrow,\ \rho\downarrow$ yields $\bar{\nu}_e, \nu_{\mu/\bar{\mu}}, \nu_{\tau/\bar{\tau}}$

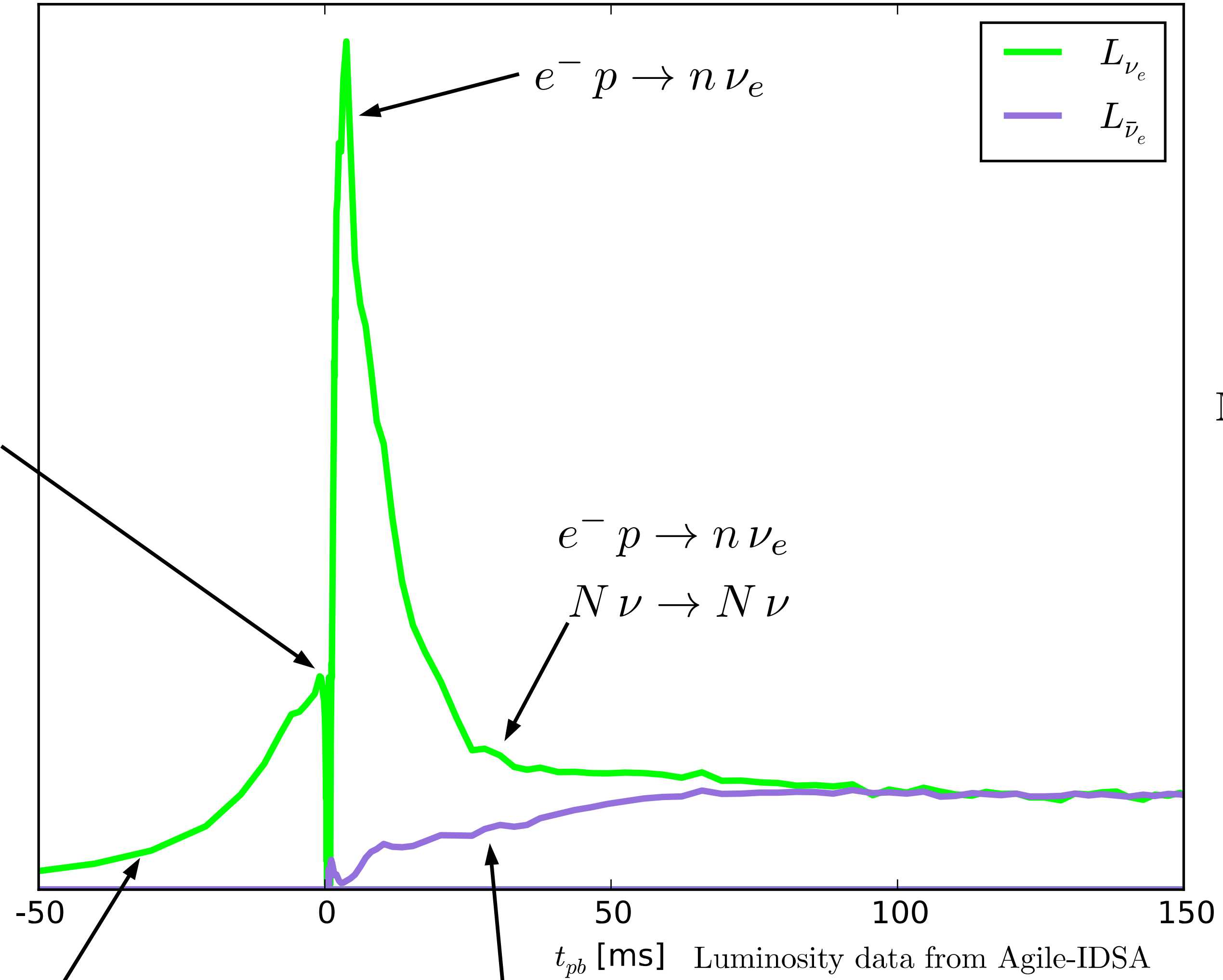
6. Neutrino absorption and continuation of neutrino-driven mechanism *(delayed shock revival)*:



Luminosity data from Agile-IDSA

- Neutrino shock revival mechanism: Wilson(1985), Bethe (1990)
  ~ 1% - 10% of neutrinos are enough to revive shock

ν-relevant Reactions



Luminosity data from Agile-IDSA

$$e^- \, p \rightarrow n \, \nu_e$$

$$\nu \, (A, Z) \rightarrow (A, Z) \, \nu$$
$$\nu \, e^\pm \rightarrow \nu \, e^\pm$$
$$N \, \nu \rightarrow N \, \nu$$

$$e^- \, p \rightarrow n \, \nu_e$$
$$N \, \nu \rightarrow N \, \nu$$

$$e^- \, (A, Z) \rightarrow \nu_e \, (A, Z-1)$$

$$e^+ \, n \rightarrow p \, \bar{\nu}_e$$
$$N \, \nu \rightarrow N \, \nu$$

More channels for all neutrinos:

$$N \, N \rightarrow N \, N \, \nu \, \bar{\nu}$$
$$e^+ \, e^- \rightarrow \nu \bar{\nu}$$
$$N \, \nu \rightarrow N \, \nu$$
$$\gamma \, \gamma \rightarrow \nu \, \bar{\nu}$$

Goal & General Strategy

Goal:

- Develop a new neutrino treatment which is simple, efficient, and reproduces most important features of neutrinos in core collapse supernova (CCSN) environment

Strategy:

- ▶ Explore CCSN dynamics with Agile-IDSA (1D, spherically symmetric GR code) using 15$M_\odot$ progenitor, Lattimer-Swesty nuclear EOS
- ▶ Use 1D profiles ($\rho,\ T,\ Y_{\rm e}$), develop our scheme to obtain $\dot{e}$ and $\dot{Y}_{\rm e}$
- ▶ Test scheme against known quantities such as $L_\nu,\ \langle\epsilon_\nu\rangle,\ \dot{e},\ \dot{Y}_{\rm e}$ for reproducibility

Code Integration:

- ▶ Replacement as a candidate for neutrino transport in hydrodynamics codes via $\dot{e}$ and $\dot{Y}_{\rm e}$
- ▶ Ultimately, be able to use scheme for long-time simulations of neutron star cooling and nucleosynthesis

Building Energy & Abundance Rates from Radial Derivatives of Luminosity

- After bounce, for $\mathrm{r} > \mathrm{r}_{\mathrm{trapping}} \equiv R_\nu$:

$$\boxed{\dot{e} = -\left[\frac{\partial L_{\nu_e}}{\partial r} + \frac{\partial L_{\bar{\nu}_e}}{\partial r}\right] \cdot \frac{1}{4\pi r^2 \rho}}$$

$$\boxed{\dot{Y}_{\mathrm{e}} = \left[\frac{\partial L^N_{\bar{\nu}_e}}{\partial r} - \frac{\partial L^N_{\nu_e}}{\partial r}\right] \cdot \frac{m_{\mathrm{b}}}{4\pi r^2 \rho}}$$

- Dominant reactions for ν_e & $\bar{\nu}_e$:

$$\begin{array}{lll}
e^- + p \longrightarrow n + \nu_e & Y_{\mathrm{e}} \downarrow & \dot{e} \downarrow \\
e^+ + n \longrightarrow p + \bar{\nu}_e & Y_{\mathrm{e}} \uparrow & \dot{e} \downarrow \\
n + \nu_e \longrightarrow e^- + p & Y_{\mathrm{e}} \uparrow & \dot{e} \uparrow \\
p + \bar{\nu}_e \longrightarrow e^+ + n & Y_{\mathrm{e}} \downarrow & \dot{e} \uparrow
\end{array}$$

- Construct infinitesimal luminosity from heating and cooling, based on Janka (2001):

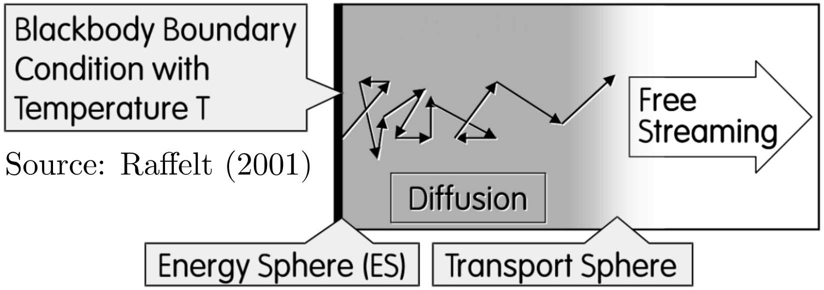
$$\frac{1}{4\pi r^2 \rho} \frac{\partial L_\nu(r)}{\partial r} = -\, q^+_\nu(r) + q^-_\nu(r)$$

$$\frac{1}{4\pi r^2 \rho} \frac{\partial L^N_\nu(r)}{\partial r} = -\, r^+_\nu(r) + r^-_\nu(r)$$

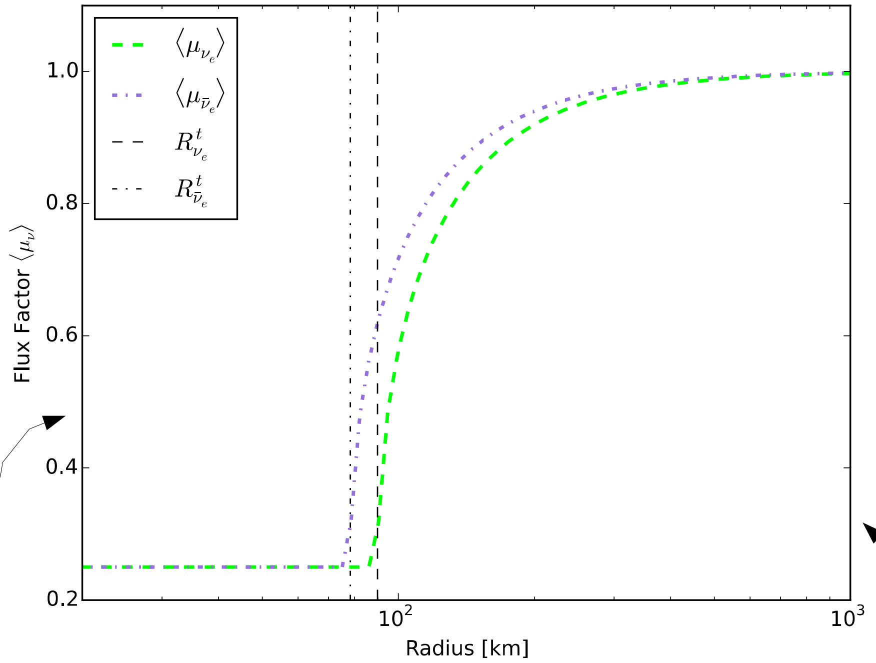
where q_ν is the specific energy rate in $[\mathrm{erg/g \cdot s}]$ and r_ν is the specific number rate in $[\#/\mathrm{g \cdot s}]$

Luminosity Estimation Scheme

New Ansatz - Based on Janka (2001):



Source: Raffelt (2001)



$$L_\nu^N(r) = L_\nu^N(R_\nu)\cdot\overbrace{\exp\left\{-\int_{R_\nu}^{r} dr'\,\frac{\langle\kappa_\nu^{\mathrm{abs}}\rangle}{\langle\mu_\nu\rangle}\right\}}^{\text{dampening}} + \int_{R_\nu}^{r} dr'\cdot 4\pi(r')^2\cdot R^-_{\nu,\mathrm{cool}}\cdot\underbrace{\exp\left\{-\int_{r'}^{r} dr''\,\frac{\langle\kappa_{\mathrm{abs}}\rangle}{\langle\mu_\nu\rangle}\right\}}_{\text{re-absorption}}$$

- Energy luminosity determined by:

$$L_\nu(r)\simeq\langle\epsilon_\nu\rangle\Big|_{r=R_\nu}\cdot L_\nu^N(r)$$

$$\langle\epsilon_\nu\rangle\Big|_{r=R_\nu} = \mathrm{T}(R_\nu)\cdot\overbrace{\frac{\mathcal{F}_3(0)}{\mathcal{F}_2(0)}}^{\sim 3.15\ldots}\simeq 3.15\ldots\cdot T(R_\nu)$$

Neutrino spheres

__Motivation: Identify the location of the boundary condition__

- Cross Section

$$\sigma_\nu \propto \sigma_o \cdot \epsilon_\nu^2 \;\rightarrow\; \kappa_\nu = \frac{\sigma_\nu \cdot \rho \cdot Y_i}{m_b}$$

- Total Opacity

$$\kappa_\nu^t = \kappa_\nu^{\text{abs}} + \kappa_\nu^{\text{scat}}$$

- Effective Opacity

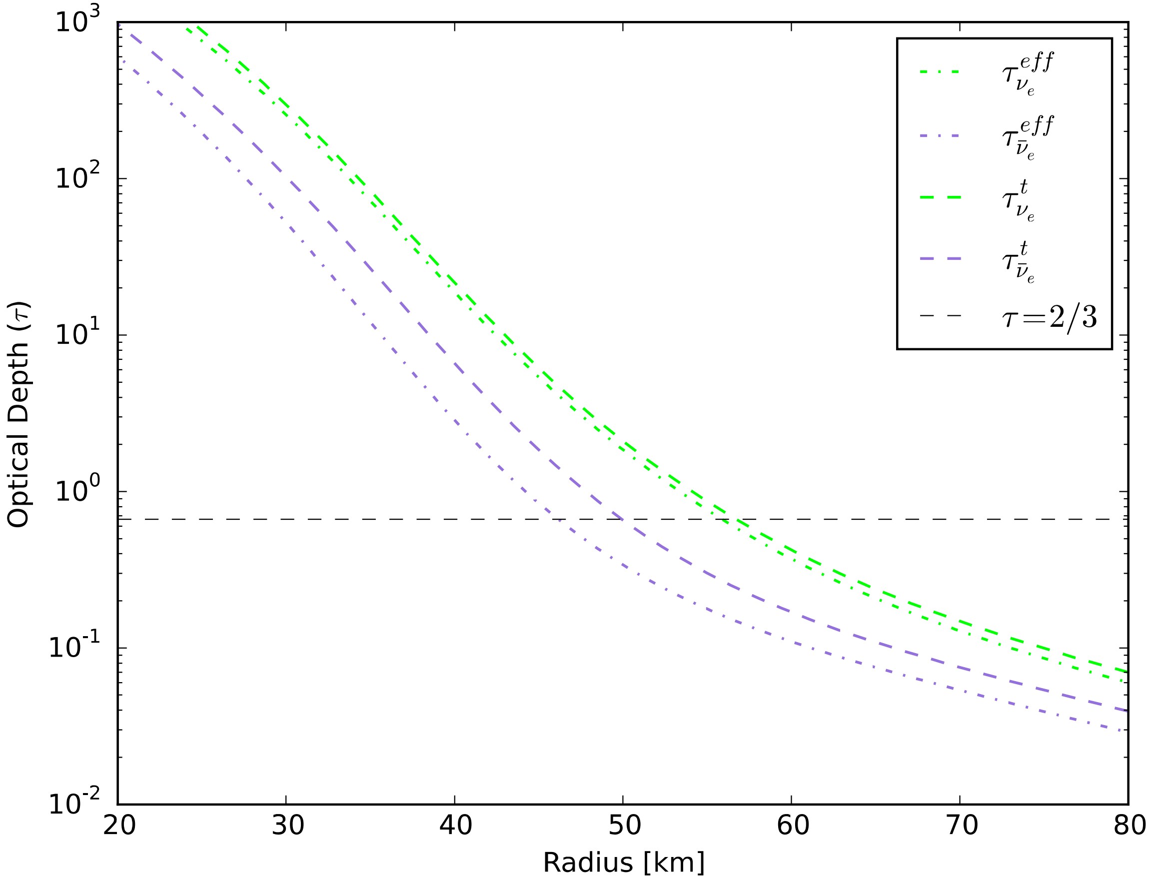
$$\kappa_\nu^{\text{eff}} = \sqrt{\kappa_\nu^{\text{abs}} \cdot \left(\kappa_\nu^{\text{abs}} + \kappa_\nu^{\text{scat}}\right)}$$

- Optical Depth
  & Neutrino spheres

$$\tau_\nu(r) = \int_r^\infty \kappa_\nu(r')\, dr'$$

$$\tau_\nu(R_\nu) = 2/3 = \int_{R_\nu}^\infty \kappa_\nu(r')\, dr'$$

- τ_ν may be physically interpreted as number of ν interactions before leaving system (i.e. neutrino at $\tau_\nu \simeq 100$ will experience ~ 100 interactions on average of given type)



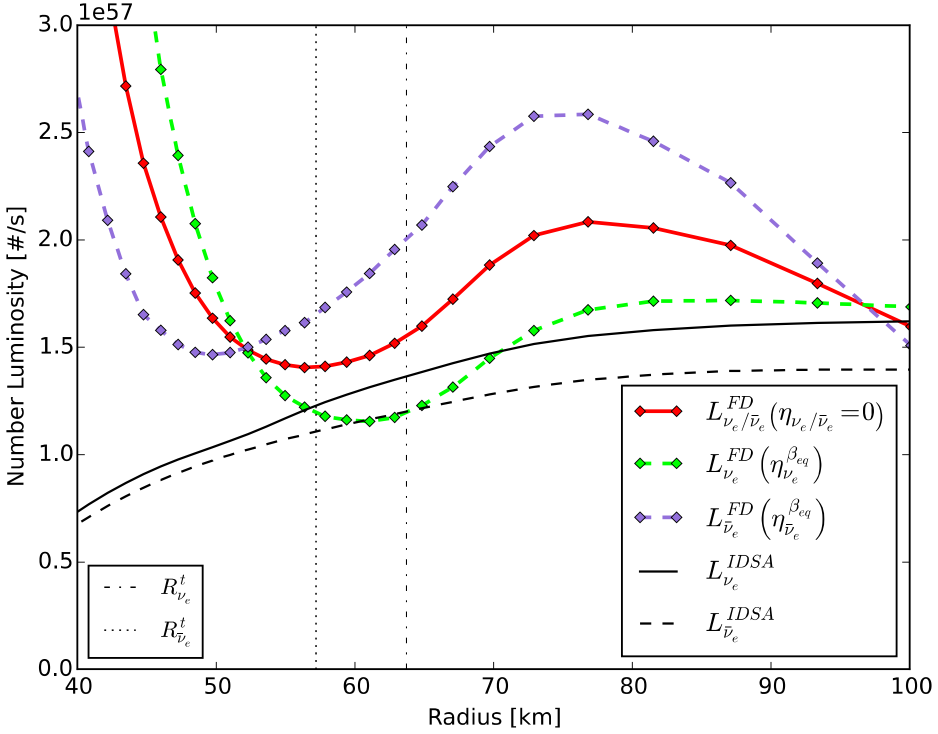
Fermi Blackbody Luminosity

Motivation: Characterize nature of and give a value to the boundary condition

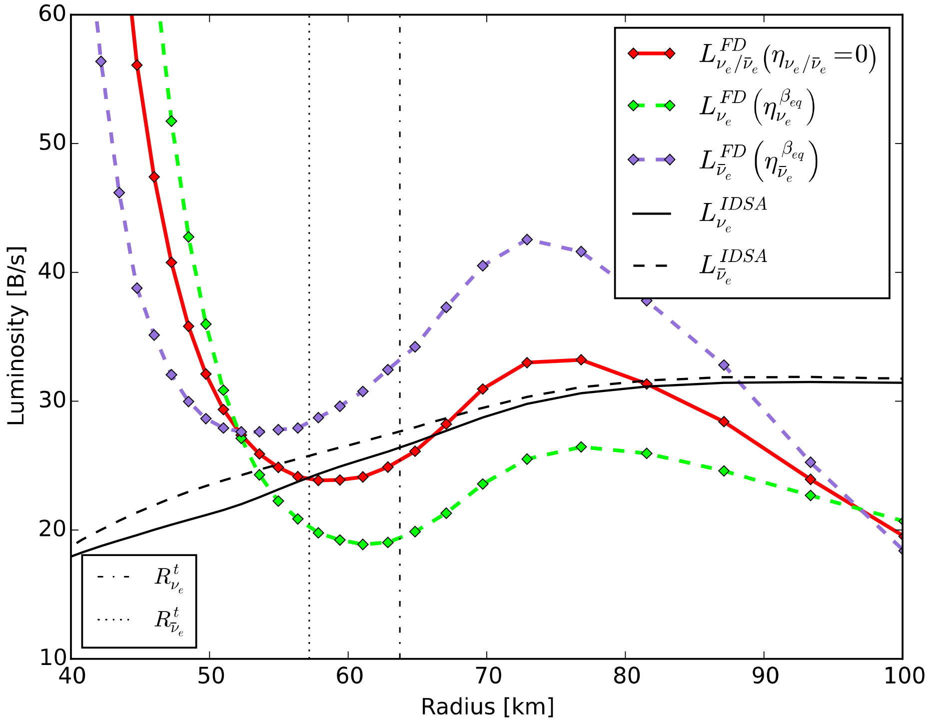
- Mimic production of neutrinos via blackbody luminosity. Near R_ν, T $\simeq T_\nu$.
- Test ν degeneracy at neutrino spheres. Which hierarchy is best assumption?

$$L_\nu^{N,FD} = \frac{4\pi^2 \cdot c}{(hc)^3} \cdot R_\nu^2 \cdot (k_B T_\nu)^3 \cdot \mathcal{F}_2(\eta_\nu)$$

$$L_\nu^{FD} = \frac{4\pi^2 \cdot c}{(hc)^3} \cdot R_\nu^2 \cdot (k_B T_\nu)^4 \cdot \mathcal{F}_3(\eta_\nu)$$



$t_{\text{pb}} \sim 300$ ms p.b.



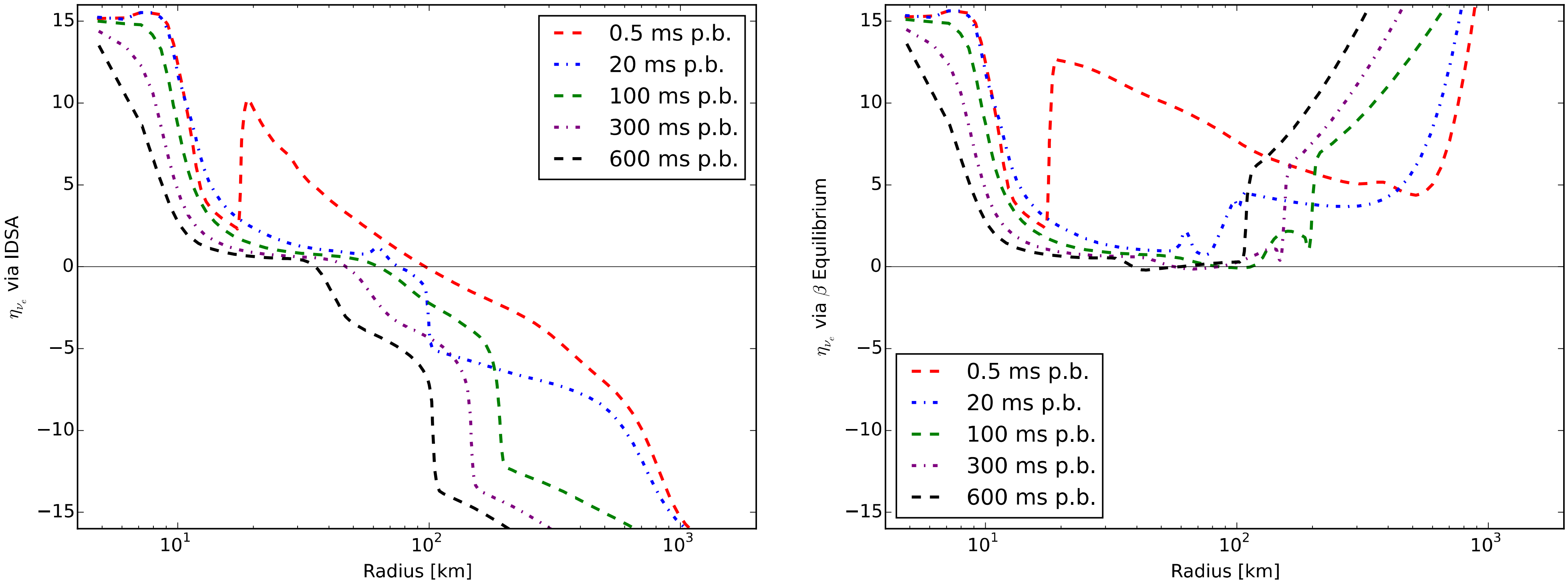
$t_{\text{pb}} \sim 300$ ms p.b.

Neutrino Degeneracy at Various Times

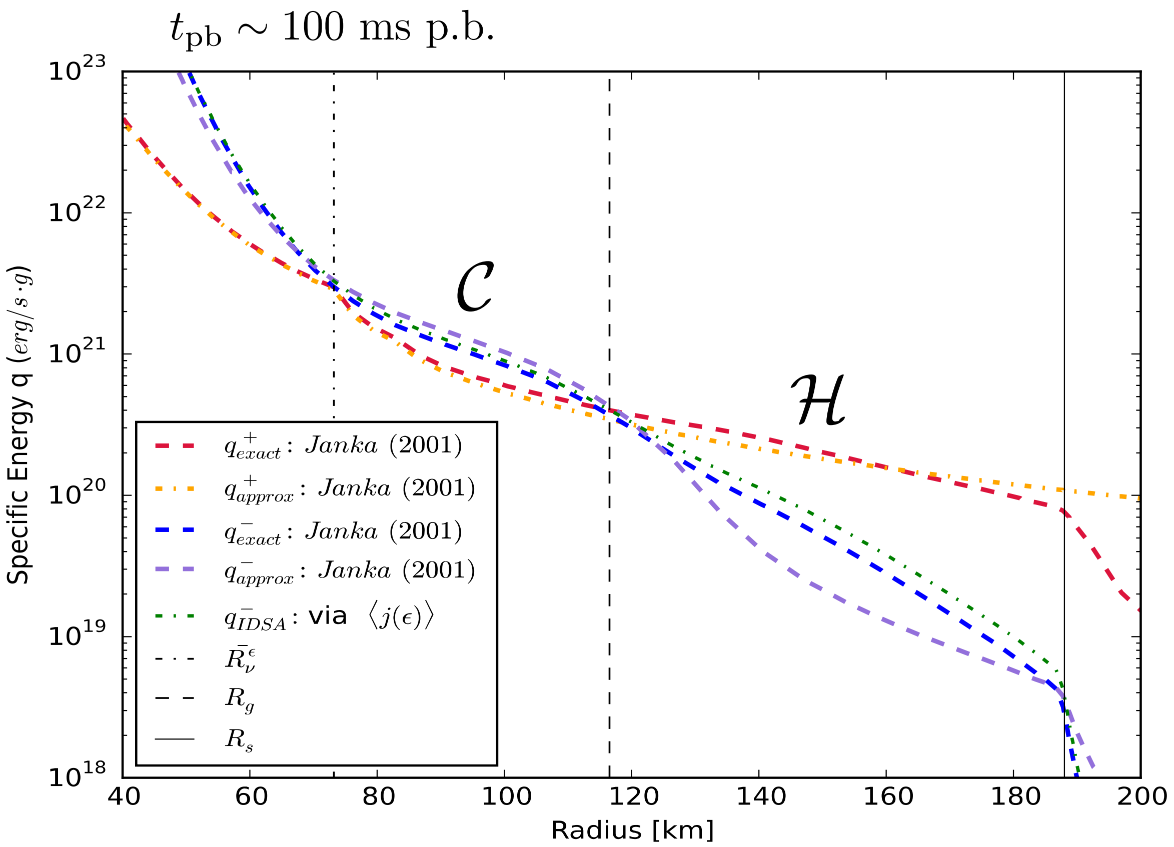
- Comparison shows excellent agreement in β equilibrium: $\rho \gtrsim 10^{11} - 10^{12}\ \mathrm{g/cm^3}$
- $\mathrm{e}^- + p \longrightarrow n + \nu_e$ yields $\mu_e + \mu_p \longrightarrow \mu_n + \mu_{\nu_e}$

$$\mu_{\nu_e} \longrightarrow +\mu_e - \hat{\mu} - \Delta_{np}, \quad \mu_{\bar{\nu}_e} = -\mu_{\nu_e}$$

- Assumption of $\eta_\nu = 0$ everywhere exclusively is insufficient for neutrino transport, but seems to be enough for Fermi blackbody boundary condition at later times! Used by Janka (2001) in: $\mathrm{R}_\nu < r < R_s$



Neutrino Heating & Cooling



- Gain radius defined as location where heating overtakes cooling:

$$R_g : q^- = q^+$$

Comparative competition of inputs:

- Cooling determined by matter temperature & ν emissivity:

$$q^-_{\rm approx} \propto (k_B T)^6$$

$$q^-_{\rm exact} \propto (k_B T)^6 \cdot [Y_p \mathcal{F}_5(\eta_e) + Y_n \mathcal{F}_5(-\eta_e)]$$

$$q^-_{\rm IDSA} = \frac{4\pi c}{(hc)^3 \rho} \int_0^\infty d\epsilon\, \epsilon^3 f_{FD}(\epsilon)\, j_{\rm IDSA}(\epsilon)$$

- Heating dependent on luminosity from re-absorption:

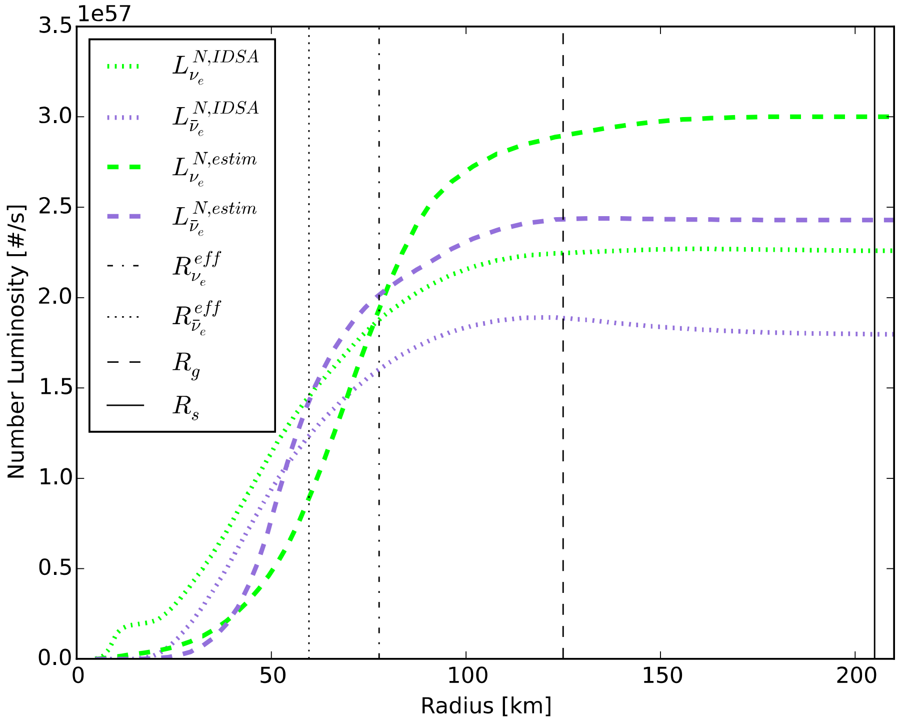
$$q^+_{\rm approx} \propto \frac{L_\nu}{r^2 \cdot \langle \mu_\nu \rangle} \cdot (k_B T_\nu)^2$$

$$q^+_{\rm exact} \propto \frac{L_\nu}{4\pi r^2 \cdot \langle \mu_\nu \rangle} \cdot \langle \epsilon_\nu^2 \rangle \cdot (Y_n + Y_p)$$

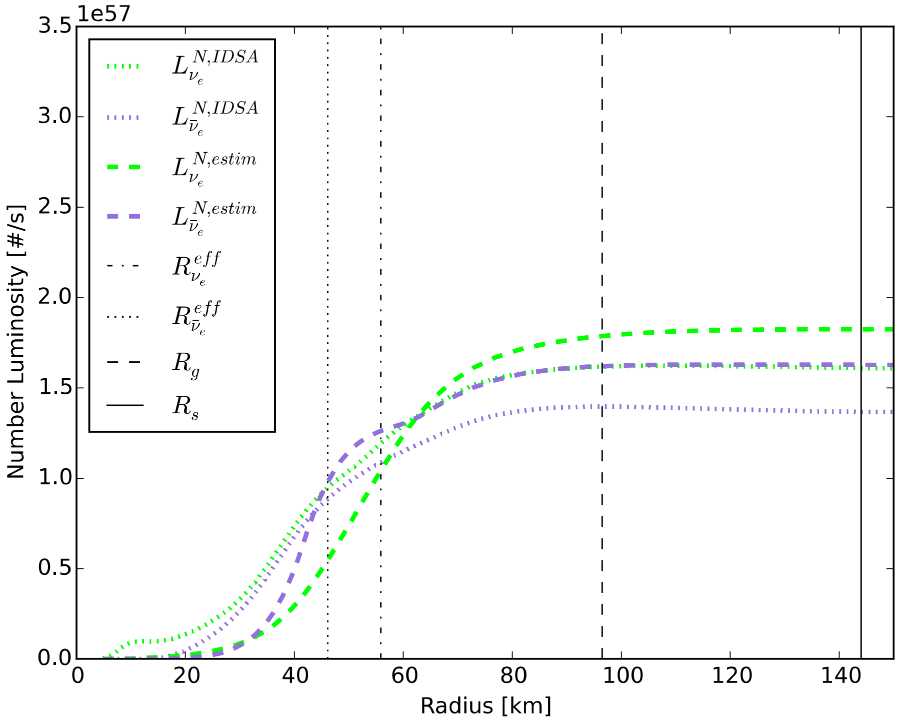
Luminosities vs. Radii: Putting it all together...

- Number luminosity at infinity is well approximated compared to IDSA.
- Diffusion prescription spreads, or smears, L_ν^{FD} into the neutrinosphere.
- Starting point determined by Fermi blackbody light bulb!
- How does it act thereafter? $\mathrm{L}_{\mathrm{acc}} = L_\nu(R_g) - L_\nu(R_\nu) \qquad L_{\mathrm{acc}} \geq 0$

$$\mathcal{H} - \mathcal{C} = L_\nu(R_g) - L_\nu(R_s) \qquad L_\nu(R_g) \geq L_\nu(R_s)$$

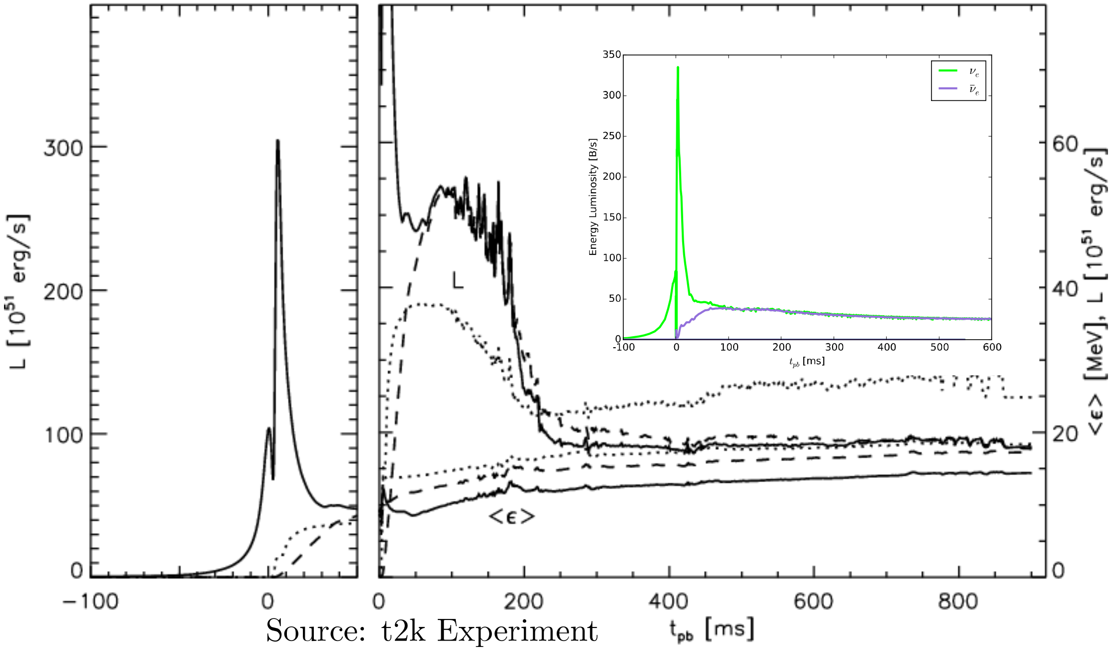


$t_{\mathrm{pb}} \sim 120$ ms p.b.

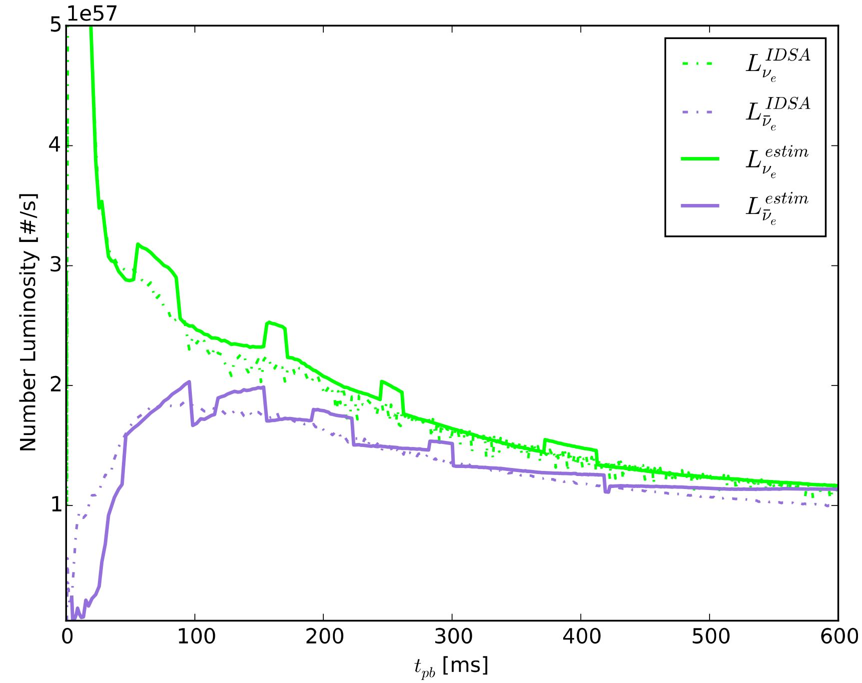


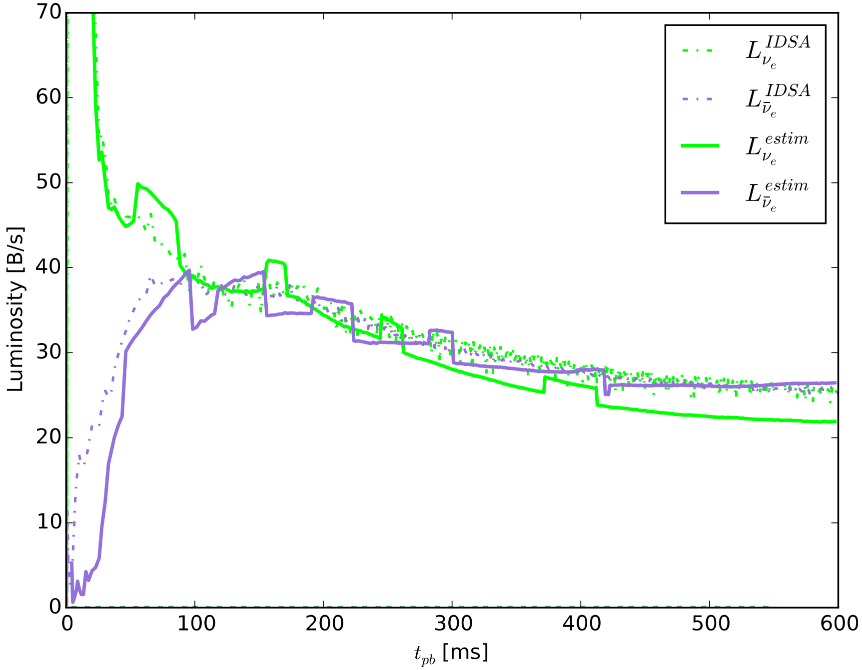
$t_{\mathrm{pb}} \sim 300$ ms p.b.

Luminosity Estimate vs. IDSA in Time



Source: t2k Experiment

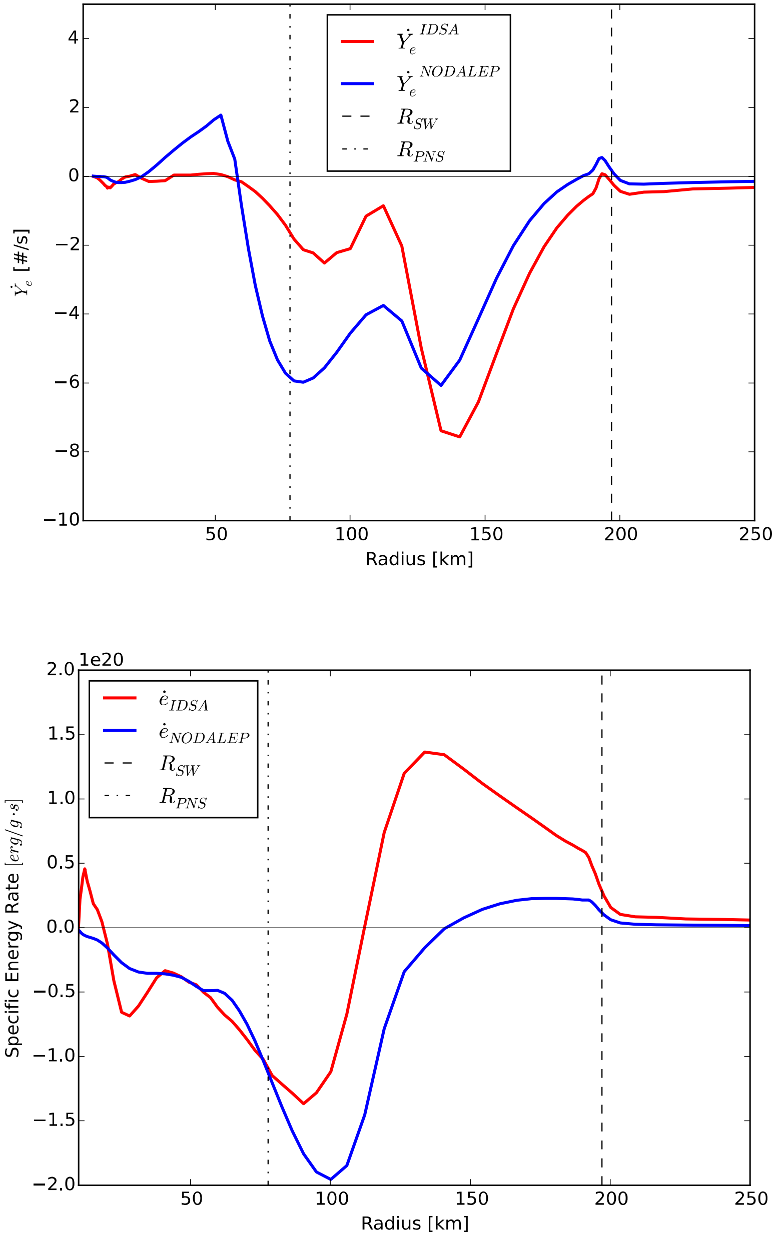




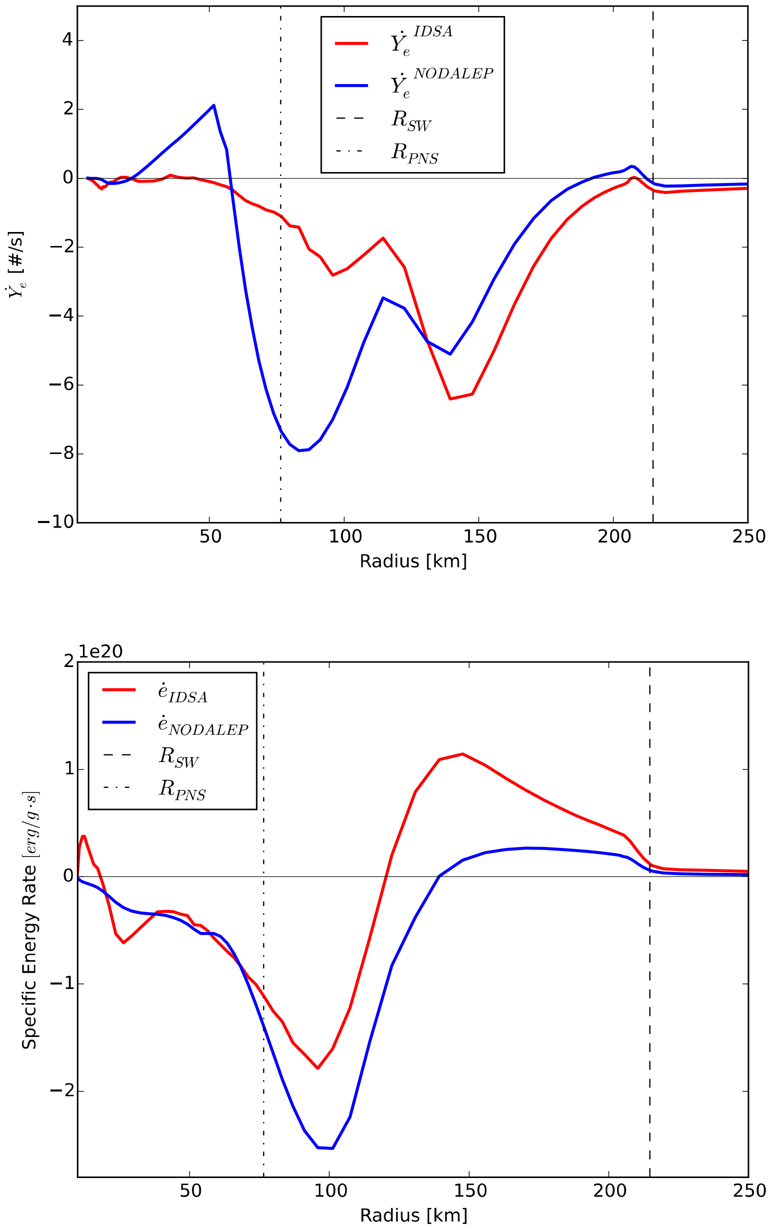
<u>Characteristic Parts of Neutrino Signal:</u>

- Prompt ν_e burst
- Stalling & Luminosity from Accretion
- Late cooling of proto-neutron star

Evolution of Energy & Abundance Change Rates ($\dot{e}$ & $\dot{Y}_\mathrm{e}$): NODALEP vs. IDSA

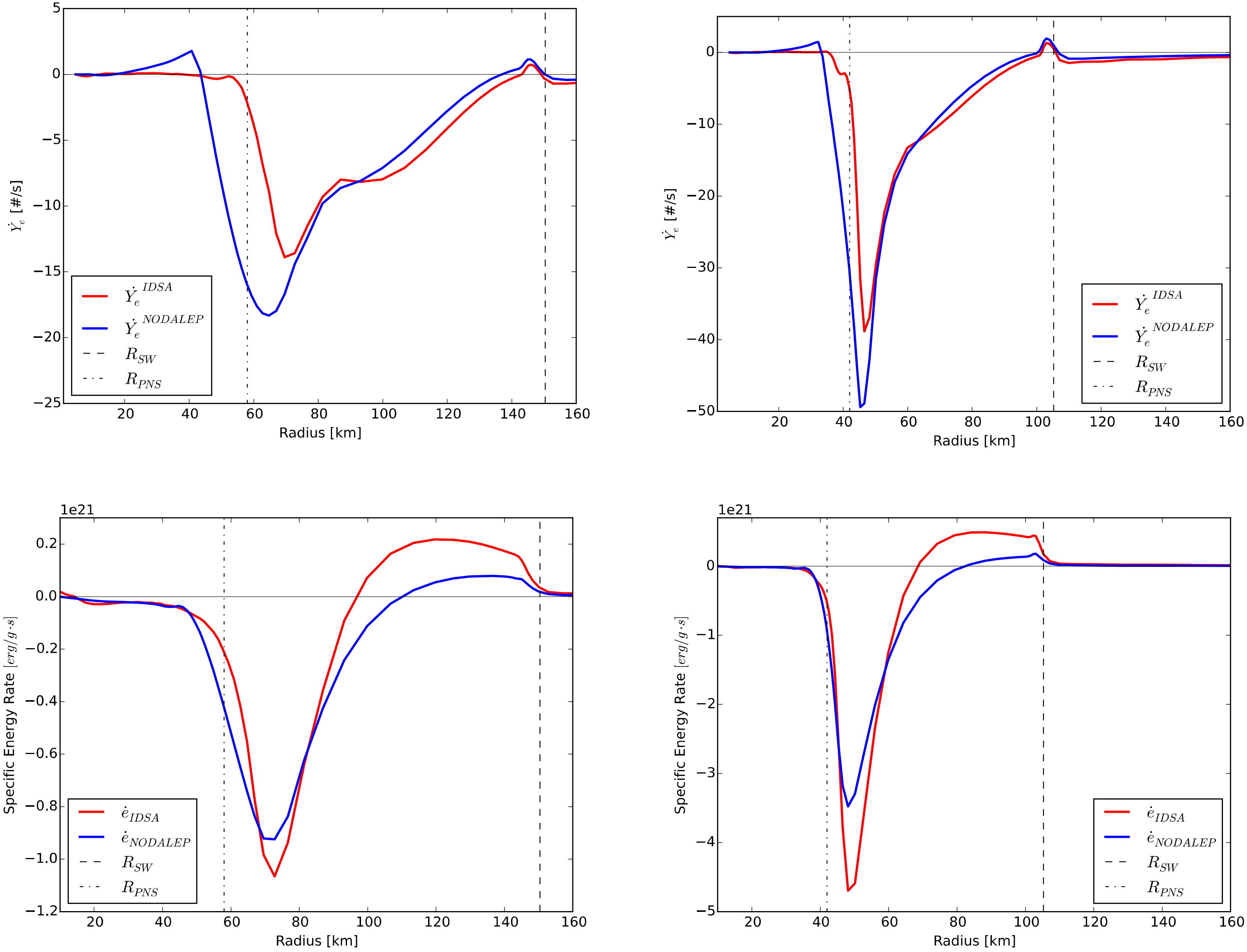


$t_\mathrm{pb} \sim 100$ ms p.b.



$t_\mathrm{pb} \sim 120$ ms p.b.

Evolution of Energy & Abundance Change Rates ($\dot{e}$ & $\dot{Y}_\mathrm{e}$): NODALEP vs. IDSA

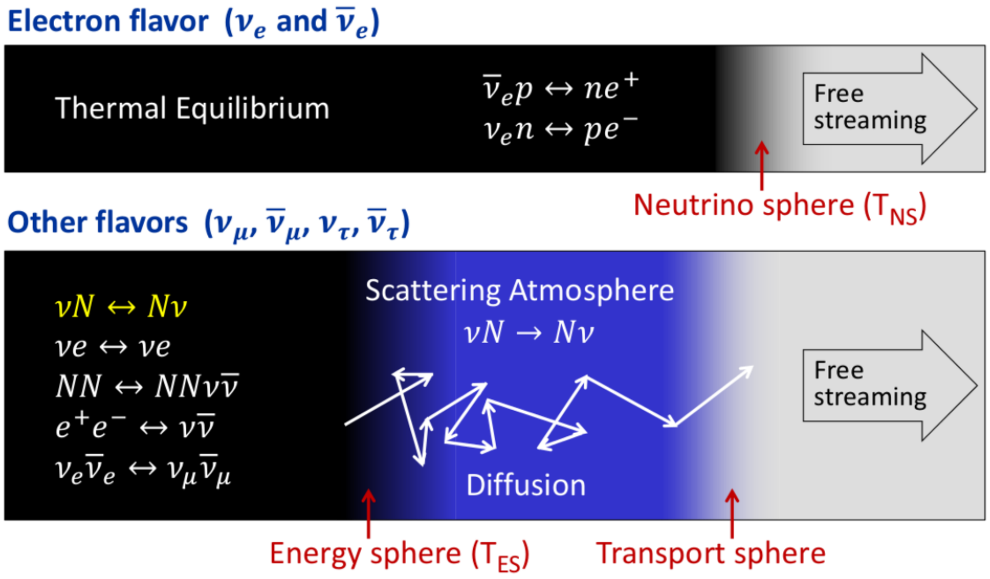


$t_\mathrm{pb} \sim 300$ ms p.b.

$t_\mathrm{pb} \sim 600$ ms p.b.

Why the difference?

- Raffelt (2012)



Electron flavor (ν_e and $\bar{\nu}_e$)

Thermal Equilibrium

$\bar{\nu}_e p \leftrightarrow n e^+$
$\nu_e n \leftrightarrow p e^-$

Free streaming

Neutrino sphere (T_{NS})

Other flavors ($\nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$)

$\nu N \leftrightarrow N \nu$
$\nu e \leftrightarrow \nu e$
$NN \leftrightarrow NN\nu\bar{\nu}$
$e^+e^- \leftrightarrow \nu\bar{\nu}$
$\nu_e\bar{\nu}_e \leftrightarrow \nu_\mu\bar{\nu}_\mu$

Scattering Atmosphere
$\nu N \rightarrow N \nu$

Diffusion

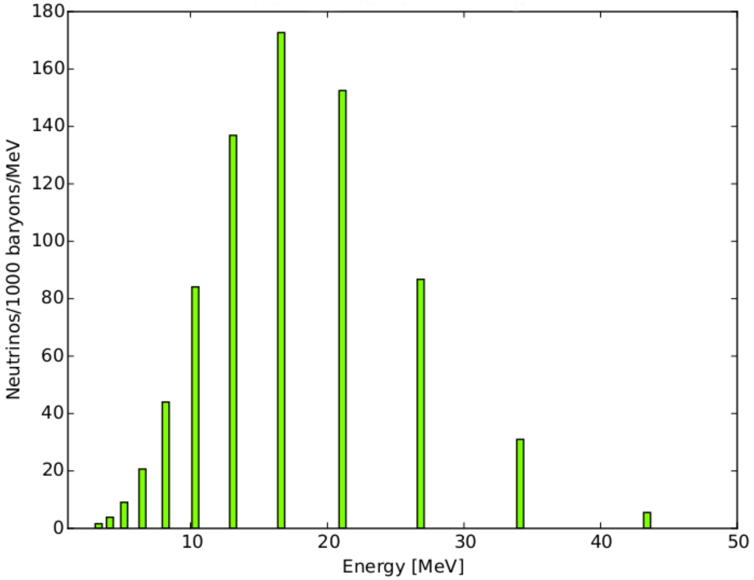
Free streaming

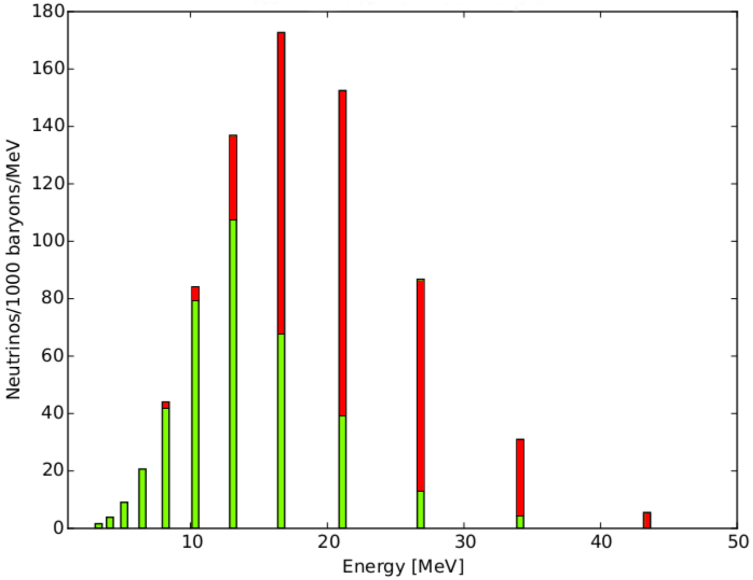
Energy sphere (T_{ES})

Transport sphere

- In our new transport, we have a superposition of ν reactions, yielding a similar "diffusive atmosphere".
- Luminosity estimates are only approximate!
- Spectral calculation would improve result.

- Spectral "clipping"

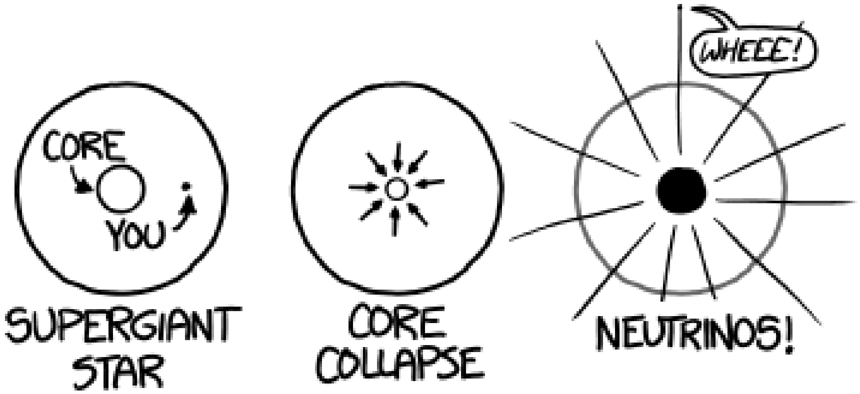




Summary & Conclusion

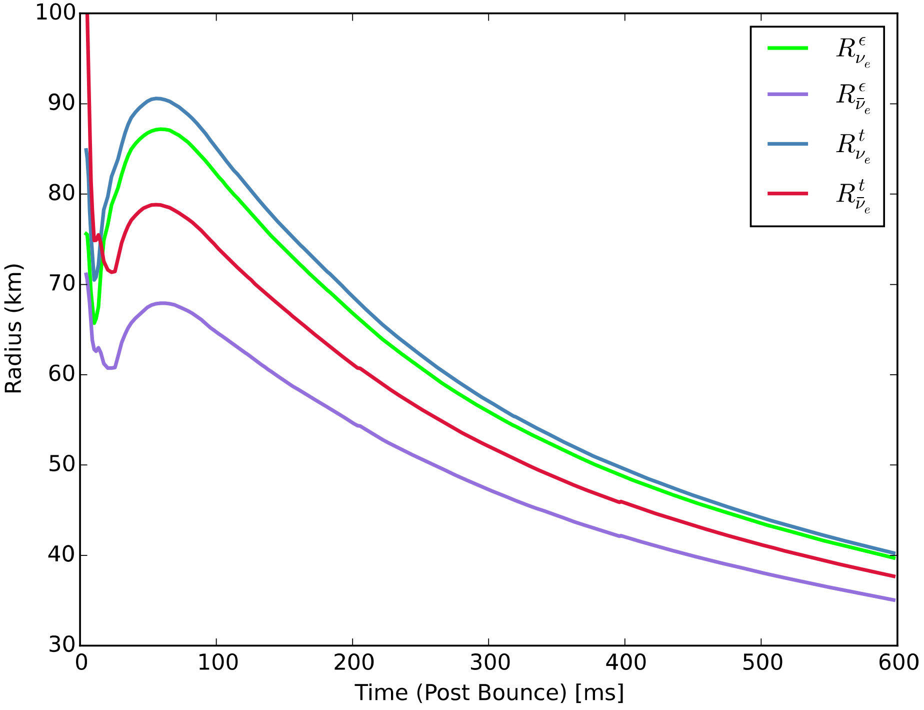
- Shock revival by neutrino heating is currently a plausible explanation for successful explosions of CCSN
- A simple, efficient neutrino treatment is demanded
- A new neutrino treatment scheme for luminosity and specific number/energy rate estimation was written and compared to output from Agile-IDSA
- Most major features of IDSA were reproduced in our neutrino treatment.
- Better treatment of neutrino degeneracy parameters at different times is needed.
- Regions between neutrino spheres contribute to greatest uncertainty
- Spectral "clipping" may motivate the need for a spectral treatment.

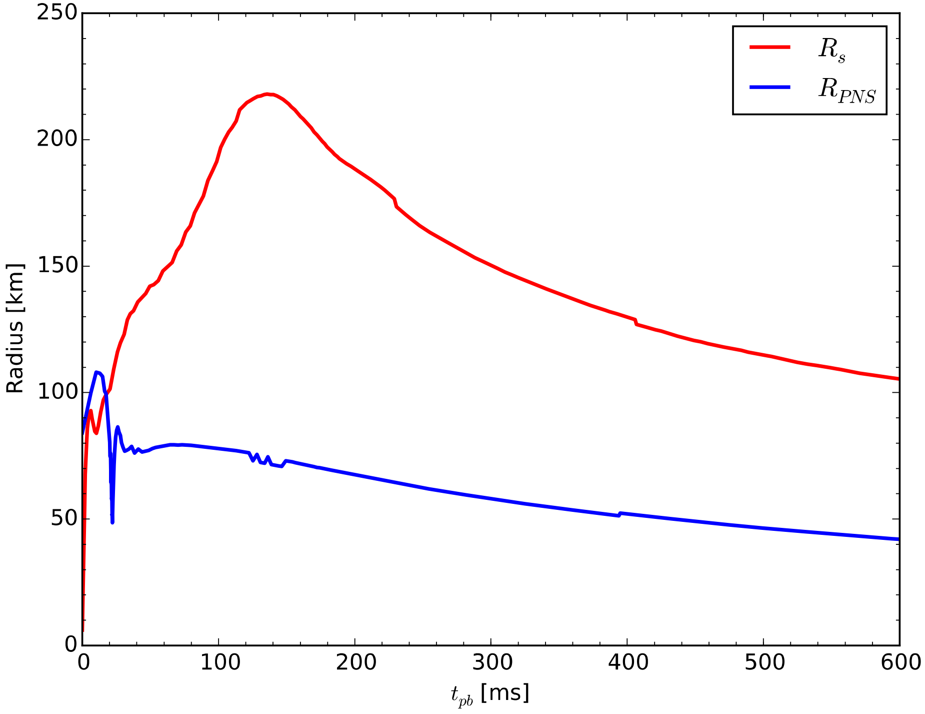
Thank you for your attention!

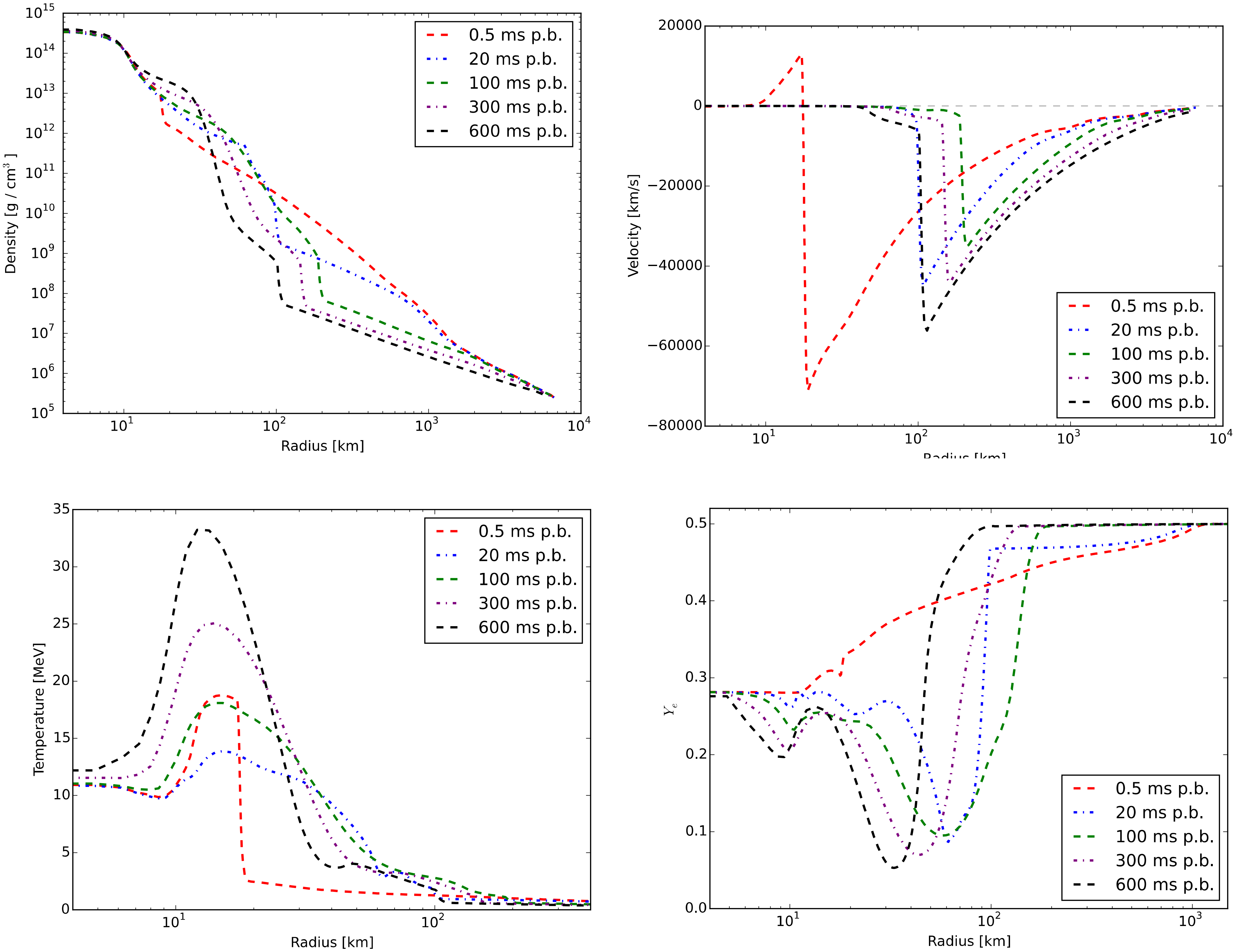


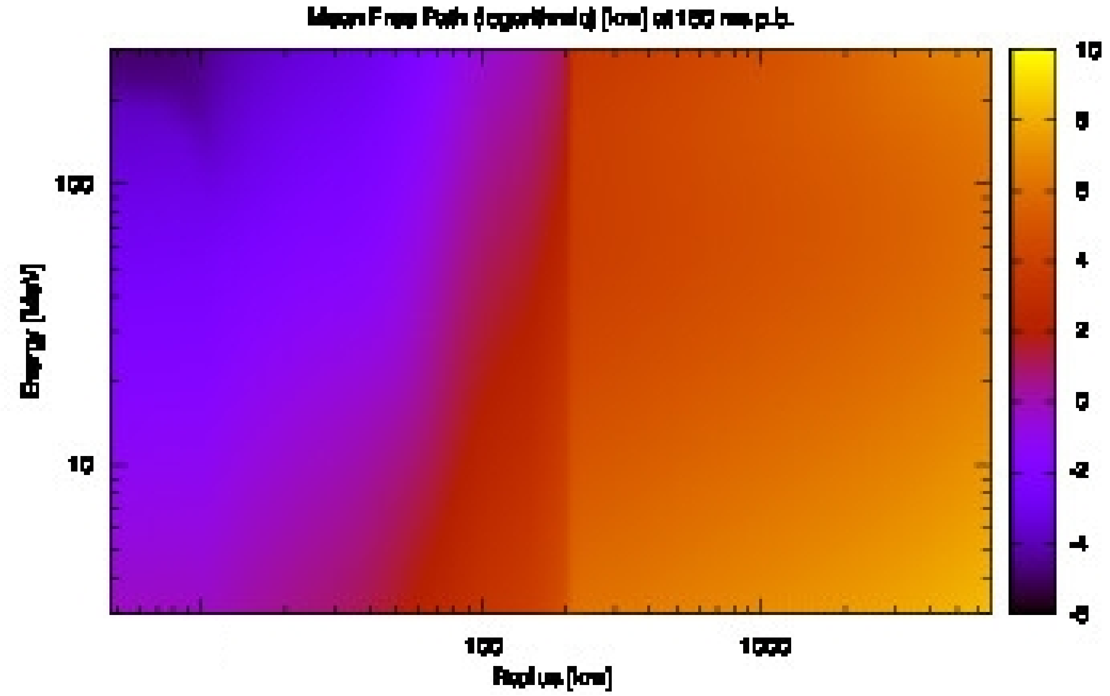
Questions?

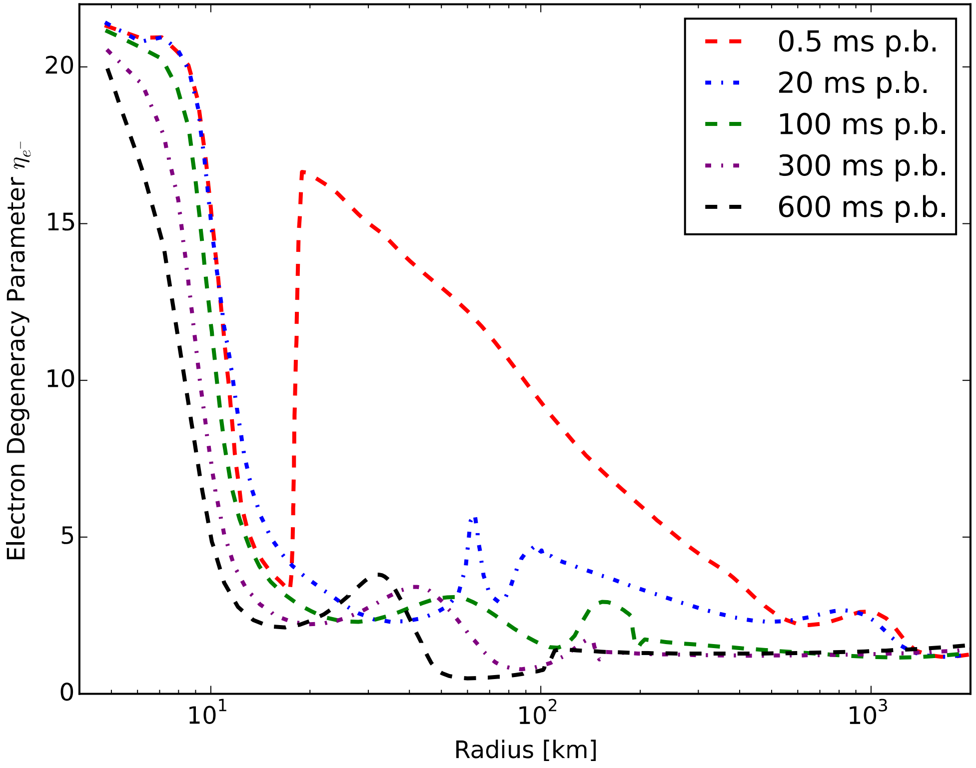
Source: xkcd


$R^{\epsilon}_{\nu_e}$
$R^{\epsilon}_{\bar{\nu}_e}$
$R^{t}_{\nu_e}$
$R^{t}_{\bar{\nu}_e}$
Radius (km)
100
90
80
70
60
50
40
30
0
100
200
300
400
500
600
Time (Post Bounce) [ms]


R_s
R_{PNS}
Radius [km]
250
200
150
100
50
0
0
100
200
300
400
500
600
t_{pb} [ms]

Density [g / cm^3]
10^{15}
10^{14}
10^{13}
10^{12}
10^{11}
10^{10}
10^9
10^8
10^7
10^6
10^5
10^1
10^2
10^3
10^4
Radius [km]
0.5 ms p.b.
20 ms p.b.
100 ms p.b.
300 ms p.b.
600 ms p.b.
Velocity [km/s]
20000
0
−20000
−40000
−60000
−80000
10^1
10^2
10^3
10^4
Radius [km]
0.5 ms p.b.
20 ms p.b.
100 ms p.b.
300 ms p.b.
600 ms p.b.
Temperature [MeV]
35
30
25
20
15
10
5
0
10^1
10^2
Radius [km]
0.5 ms p.b.
20 ms p.b.
100 ms p.b.
300 ms p.b.
600 ms p.b.
Y_e
0.5
0.4
0.3
0.2
0.1
0.0
10^1
10^2
10^3
Radius [km]
0.5 ms p.b.
20 ms p.b.
100 ms p.b.
300 ms p.b.
600 ms p.b.


Mean Free Path (logarithmic) [km] at 100 ms p.b.
Energy [MeV]
100
10
Radius [km]
100
1000
10
8
6
4
2
0
-2
-4
-6


Electron Degeneracy Parameter η_{e^-}
20
15
10
5
0
Radius [km]
10^1
10^2
10^3
0.5 ms p.b.
20 ms p.b.
100 ms p.b.
300 ms p.b.
600 ms p.b.